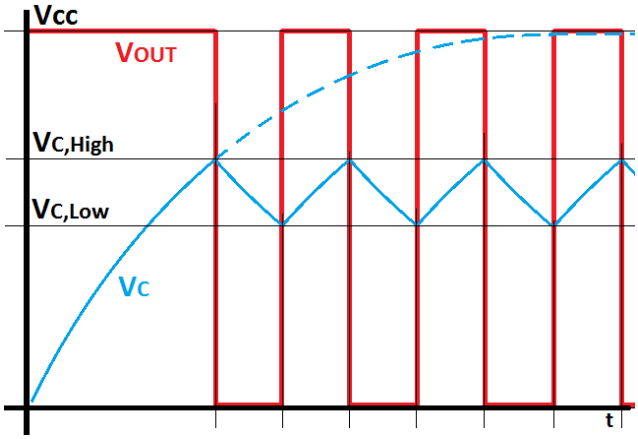
RC-Oscillator frequency calculation



Charge:

$$V_C = V_{CC}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$1 - \frac{V_C}{V_{CC}} = e^{-\frac{t}{RC}}$$

$$ln\left(1 - \frac{V_C}{V_{CC}}\right) = -\frac{t}{RC}$$

$$RC \cdot -ln\left(1 - \frac{V_C}{V_{CC}}\right) = t$$

$$t_{charge} = t_{high} - t_{low} = RC \cdot -ln\left(1 - \frac{V_{C\,high}}{V_{CC}}\right) - RC \cdot -ln\left(1 - \frac{V_{C,low}}{V_{CC}}\right)$$

$$= RC\left(ln\left(1 - \frac{V_{C,low}}{V_{CC}}\right) - ln\left(1 - \frac{V_{C,high}}{V_{CC}}\right)\right) = RC \cdot ln\left(\frac{1 - \frac{V_{C,low}}{V_{CC}}}{1 - \frac{V_{C,high}}{V_{CC}}}\right)$$

$$= RC \cdot ln\left(\frac{V_{CC} - V_{C,low}}{V_{CC} - V_{C,high}}\right)$$

Discharge:

$$V_C = V_{CC} \cdot e^{-\frac{t}{RC}}$$

$$\frac{V_C}{V_{CC}} = e^{-\frac{t}{RC}}$$

$$-ln\left(\frac{V_C}{V_{CC}}\right) = \frac{t}{RC}$$

$$RC \cdot -ln\left(\frac{V_C}{V_{CC}}\right) = t$$

$$t_{discharge} = t_{low} - t_{high} = RC \cdot -ln\left(\frac{V_{C,low}}{V_{CC}}\right) - RC \cdot -ln\left(\frac{V_{C,high}}{V_{CC}}\right) = RC\left(ln\left(\frac{V_{C,high}}{V_{CC}}\right) - ln\left(\frac{V_{C,low}}{V_{CC}}\right)\right)$$

$$= RC \cdot ln\left(\frac{\frac{V_{C,high}}{V_{CC}}}{\frac{V_{C,low}}{V_{CC}}}\right) = RC \cdot ln\left(\frac{V_{C,high}}{V_{C,low}}\right)$$

Frequency:

$$f = \frac{1}{t_{charge} + t_{discharge}} = \frac{1}{RC \cdot ln\left(\frac{V_{CC} - V_{C,low}}{V_{CC} - V_{C,high}}\right) + RC \cdot ln\left(\frac{V_{C,high}}{V_{C,low}}\right)}$$

$$= \frac{1}{RC \cdot ln\left(\frac{V_{CC} - V_{C,low}}{V_{CC} - V_{C,high}} \cdot \frac{V_{C,high}}{V_{C,low}}\right)}$$

Insert given Values:

$$V_{CC} = 10V$$

$$V_{C,low} = 3.9V$$

$$V_{C,high} = 5.9V$$

$$R = 100k\Omega$$

$$C = 100nF$$

$$f = \frac{1}{RC \cdot ln\left(\frac{V_{CC} - V_{C,low}}{V_{CC} - V_{C,high}} \cdot \frac{V_{C,high}}{V_{C,low}}\right)} = \frac{1}{RC \cdot 0.81} = \underline{\underline{123Hz}}$$